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Technical Report

A canopy energy balance model for the water balance model WaSiM

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Friday 9th June, 2017

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1 Canopy Energy Balance Model for WaSiM

The implemented canopy energy balance (CEB) is a so called single layer canopy energy balance model (good overview in GOUTTEVIN ET AL. (2015)). This means that the canopy structure is simplified regarded as a single layer above the ground featuring certain physical properties. The layer is supposed to be in full physical contact with the overlaying air mass. Energy is exchanged in both directions via radiation and different forms of heat fluxes. The connection to the ground or respectively the snow cover is less physically based. The sub-canopy air conditions are computed within the new "Snow Canopy Model (SCN)" introduced by FÖRSTER ET AL. (2018) using a conceptual approach. The optical properties of the canopy layer alter the radiation balance for the underlying snow cover. Of special interest is the canopy temperature T_c, which is used as the variable during the iterative solving procedure for the canopy energy balance. The resulting canopy temperature determines the amount of emitted longwave radiation from the canopy into the snow cover. This can lead to significant alterations in the snow cover energy balance and therefore to considerable changes in the snow ablation regime of the snow cover as will be shown in section 2.

The energy balance equation for the canopy layer which is solved in the newly implemented "Canopy energy balance" model is:

$$0 = SW_{net} + LW_{net}[T_{can}] + H[T_{can}] + LE[T_{can}] + A[T_{can}] + BM[T_{can}] \quad (1)$$

with:

- SW_{net} : net shortwave radiation $[W/(m^2)]$
- LW_{net} : net longwave radiation $[W/(m^2)]$
- H : sensible heat flux $[W/(m^2)]$
- LE : latent heat flux $[W/(m^2)]$
- A : advective energy flux by precipitation $[W/(m^2)]$
- BM : biomass heat storage $[W/(m^2)]$

Except for the net shortwave radiation every term is a function of the canopy temperature. The following section describes the implemented formulations for the computation of the different heat fluxes of the canopy energy balance.

1.1 Shortwave Radiation

TACONET ET AL. (1986) give a formulation for the shortwave radiation balance of the canopy:

$$SW_{net} = SW_{in}(1 - \alpha_{can})\sigma_f \Big(1 + \frac{\alpha_{surf}(1 - \sigma_f)}{1 - \sigma_f \alpha_{surf} \alpha_{can}}\Big) \tag{2}$$

with:

$SW_{net,can}$: net shortwave radiation of canopy layer $[W/(m^2K)]$
SW_{in} : incoming shortwave radiation (global Radiation) $[W/(m^2K)]$
σ_f : absorption factor $[-]$
α_{can} : albedo value of the canopy $[-]$
α_{surf} : albedo value of the ground surface $[-]$

The albedo value of the canopy is calculated in the SCN module. The absorption factor σ_f describes the amount of absorbed radiation by the canopy given by a formulation of the Beer-Lambert law:

$$\sigma_f = 1 - e^{-k_{LAI} LAI} \tag{3}$$

with:

LAI : leaf area index $[m^2/m^2]$
k_{LAI} : extinction parameter (normally between 0.4 and 0.8) $[-]$

1.2 Longwave Radiation

Assuming an emissivity value of 1, according to TACONET ET AL. (1986) the net longwave radiation balance of the canopy layer can be calculated with:

$$LW_{net} = \sigma_f(LW_{in} + \sigma T_{surf}^4 - 2\sigma T_{can}^4) \tag{4}$$

with:

$LW_{net,can}$: net longwave radiation of canopy layer $[W/(m^2K)]$

σ_f : absorption factor $[-]$

LW_{in} : incoming longwave radiation $[W/(m^2K)]$

σ : Stefan-Boltzmann constant $5.67 * 10^-8$ $[W/(m^2K^-4)]$

T_{surf} : ground surface Temperature $[K]$

T_{can} : canopy layer temperature $[K]$

1.3 Sensible Heat

For the computation of the sensible heat flux between the top-of-canopy air mass and the canopy layer a bulk transfer formulation of the sensible heat flux is applied (e.g. ANDERSON (1968), BRAITHWAITE (1995)). In this approach the turbulent energy exchange processes caused by wind are described by the use of the aerodynamic resistance of the canopy r_a. The bulk transfer coefficient C_h takes atmospheric stability conditions into account.

$$H = \frac{C_h \rho_a c_p}{r_a}(T_a - T_{can}) \tag{5}$$

$$C_{hn} = k^2 \cdot (ln(z_a/z_0))^{-2} \tag{6}$$

with:

H : sensible heat flux $[W/m^2]$

ρ_a : air density $[m/s^2]$

c_p : specific heat capacity of air $[J/(kgK)]$

C_h : bulk transfer coefficient $[-]$

r_a : aerodynamic resistance $[s/m]$

T_a : air temperature $[K]$

T_{can} : canopy temperature $[K]$

C_{hn} : bulk transfer coefficient under neutral stability conditions $[-]$

k : Karman's constant $[-]$

z_a : wind speed measurement height $[m]$

z_0 : roughness length $[m]$

The stability conditions dependent on the air temperature and the temperature of the canopy. The conditions are stable, if the canopy is significantly cooler than the air. Hence during unstable conditions the canopy is warmer than overlaying air mass, causing much stronger turbulences. In case the temperatures of the canopy and the air are almost equal neutral stability conditions prevail. For the calculation of the correct value of the bulk transfer coefficient the atmospheric stability conditions have to be assessed first. Following the equations given by OKE (1988) the bulk transfer coefficient can be assessed using the dimensionless bulk Richardson number Ri_b.

According to DEWALLE & RANGO (2008) Ri_b can be calculated with:

$$Ri_b = \frac{2g}{T_a - T_{can}} \cdot \frac{z_0(T_a - T_{can})}{u_a^2} \tag{7}$$

with:

Ri_b : Richardson number $[-]$
g : gravitational acceleration $[m/s^2]$
T_a : air temperature $[K]$
T_{can} : canopy temperature $[K]$
z_0 : roughness length $[m]$
u_a : wind speed $[m/s]$

For stable conditions C_h is calculated with:

$$C_h/C_{hn} = (1 - 5Ri_b)^2 \tag{8}$$

and for unstable conditions C_h becomes:

$$C_h/C_{hn} = (1 - 16Ri_b)^{0.75} \tag{9}$$

1.3.1 Aerodynamic Resistance

Following the methodology proposed by IRMAK & MUTIIBWA (2010) the aerodynamic resistance r_a is calculated with:

$$r_a = \frac{ln(\frac{z_m - d}{z_{0m}})\, ln(\frac{z_h - d}{z_{0h}})}{k^2 u_a} \tag{10}$$

where u_a is the wind speed at height z_m and z_h is the height of the humidity measurement. According to BRUTSAERT (1982) d, z_{0m} and z_{0h} are:

$$d = 0.67h \tag{11}$$

$$z_{0m} = 0.123h \tag{12}$$

$$z_{0h} = 0.1 z_{0m} \tag{13}$$

1.4 Latent Heat

The amount of energy transfered by vaporization and condensation of water is modeled with:

$$LE = \frac{0.622L}{R_a T_{air}} \frac{1}{r_E} C_e(e_{sat}[T_{can}] - e_{air}) \tag{14}$$

with:

L	: heat of vaporization or condensation $[J/(kgK)]$
R_a	: specific gas constant for air $[J/(kgK)]$
r_E	: effective aerodynamic resistance $[s/m]$
$e_{sat}[T_{can}]$	: saturation vapor pressure at canopy temperature $[Pa]$
e_{air}	: air vapor pressure $[Pa]$

1.4.1 Effective aerodynamic Resistances

Like in GOUTTEVIN ET AL. (2015) the effective aerodynamic resistance r_E is parametrized using the transpiration resistance r_{Etr} and the resistance against interception evaporation r_{Eint} weighted with the fraction of wet canopy:

$$\frac{1}{r_E} = \frac{1}{r_{Etr}}(1 - f_{wet}) + \frac{1}{r_{Etr}} f_{wet} \tag{15}$$

The wet fraction of the canopy is determined by the fraction of the amount of intercepted water I and the interception capacity I_{max} assessed in the SCN module:

$$f_{wet} = (I/I_{max})^{2/3} \tag{16}$$

The values of r_{Etr} and r_{Eint} have to be set in the WaSiM landuse table.

1.5 Precipitation Heat

The last heat flux that is considered in the canopy energy balance is the advective energy flux by precipitation to the canopy A. Following the approach of WARSCHER (2014) for the computation of the precipitation heat for the energy balance of the snow cover the precipitation heat is:

$$A = c_w \frac{P_{rain}}{\Delta t}(T_{air} - T_{can}) + c_{ice} \frac{P_{snow}}{\Delta t}(T_{air} - T_{can}) \tag{17}$$

c_w and c_{ice} are the specific heat capacities for water and ice respectively

1.6 Biomass Heat Storage

According to GOUTTEVIN ET AL. (2015) the amount of energy stored within the biomass of the canopy layer between two time steps BM_{can} can be accounted for by calculating:

$$BM_{can} = HM_{can} \cdot \frac{T_{can}^{t} - T_{can}^{t-1}}{\Delta t} \tag{18}$$

T_{can}^{t} and T_{can}^{t-1} are the canopy temperatures at the current and the preceding time step. The heat mass of the canopy layer HM_{can} is made up of the heat mass of leaves HM_{leaves} and the heat mass of the trunks HM_{trunk}:

$$HM_{leaves} = LAI \cdot d_{leaf} \cdot \rho_{biomass} \cdot C_{pbiomass} \tag{19}$$

$$HM_{trunk} = 0.5 \cdot B \cdot z_{can} \cdot \rho_{biomass} \cdot C_{pbiomass} \tag{20}$$

with:

d_{leaf}	: leaf Thickness $[m]$
$\rho_{biomass}$	: density of biomass $[kg/m^3]$
$C_{pbiomass}$	: biomass specific heat mass $[J/(kgK)]$
B	: stand basal area $[m^2/m^2]$
z_{can}	: canopy height $[m]$

1.7 Iteration Procedure

The variable for solving the canopy energy balance is the canopy temperature T_c. Equation 1 is solved by applying the very robust bisection method (see for example: FREUND & HOPPE (2007)). Figure 1 displays the average number of iterations needed in order to solve the canopy energy balance. The search for the solution of the CEB is restricted to the interval between 200 and 340 K. The tolerance ϵ for solving the balance equation is set to 10^{-8} K. In order to prevent unphysical solutions in case of bad input data the maximum temperature difference between the canopy and the air temperature is set to 30 K.

For this application the CEB model mostly converges within less then 10 iteration steps.

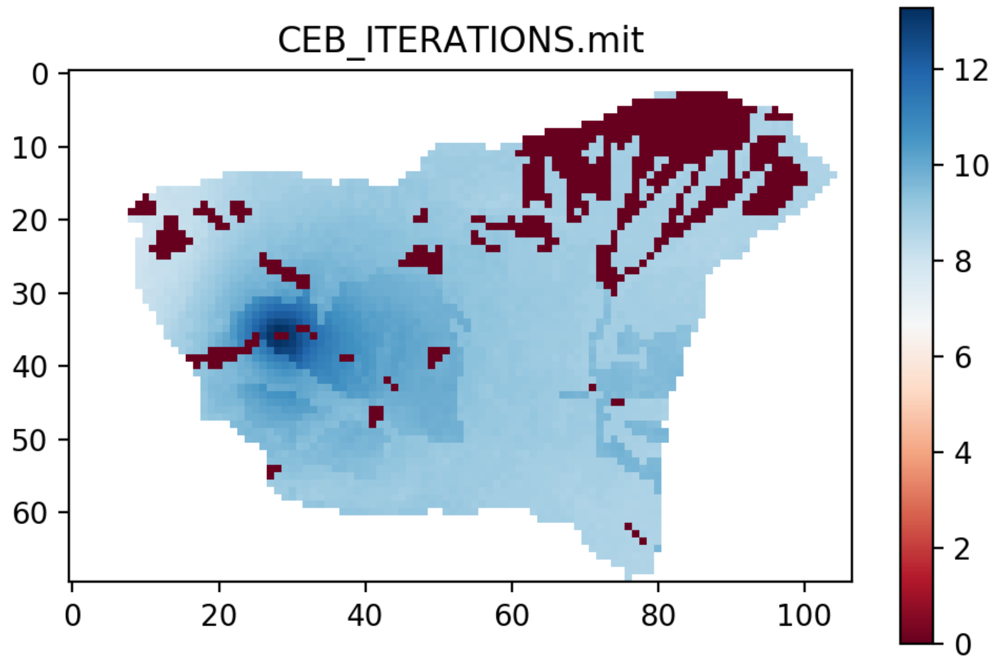

Figure 1: Average number of iterations for the calculation of the canopy energy balance in the Dreisaeulerbach catchment during winter 2017/18

2 Simulation Results

In order to show the impact of the CEB on the snow storage development three different models are compared.

- 1: No SCN + no CEB
- 2: SCN + no CEB
- 3: SCN + CEB

All models use the energy balance method to model the snow cover (WARSCHER (2014)). Each model has been calibrated using a mixed objective function taking into account discharge data as well as SWE measurements from the catchment for the winter season 2016/17. Figure 2 displays the differences in the specific storage of snow water equivalent (SWE) within the Dreisaeulerbach catchment during the validation period in the winter season 2017/18.

Neglecting the snow interception in the canopy (model 1) leads to an overestimation of stored SWE within the mostly forested catchment. Model 2 and 3 show a similar development of the snow storage

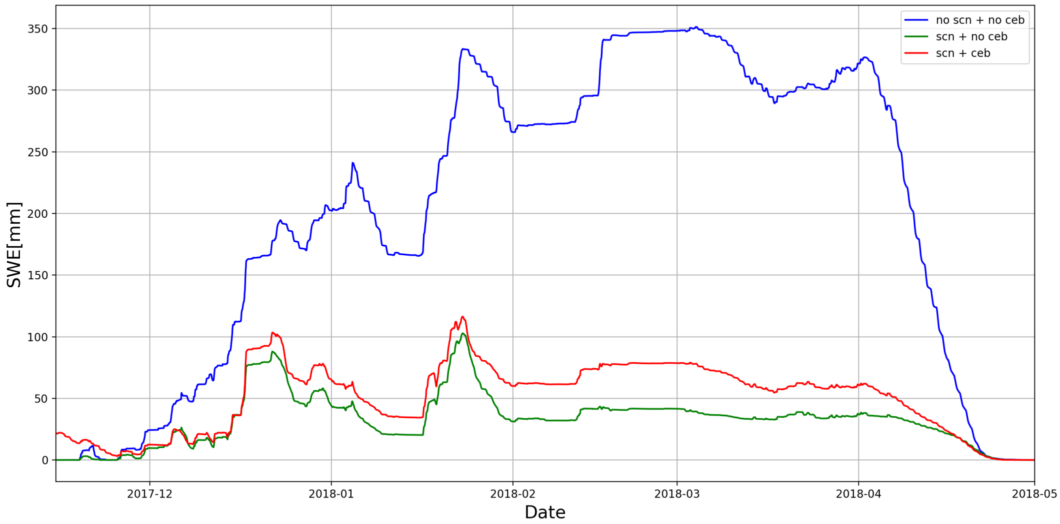

Figure 2: Specific SWE stored in the catchment

during the winter season. Nevertheless during February and March differences of up to 30 mm of SWE can be observed. Figure 3 shows the resulting discharge behaviour during the Winter season 2017/18.

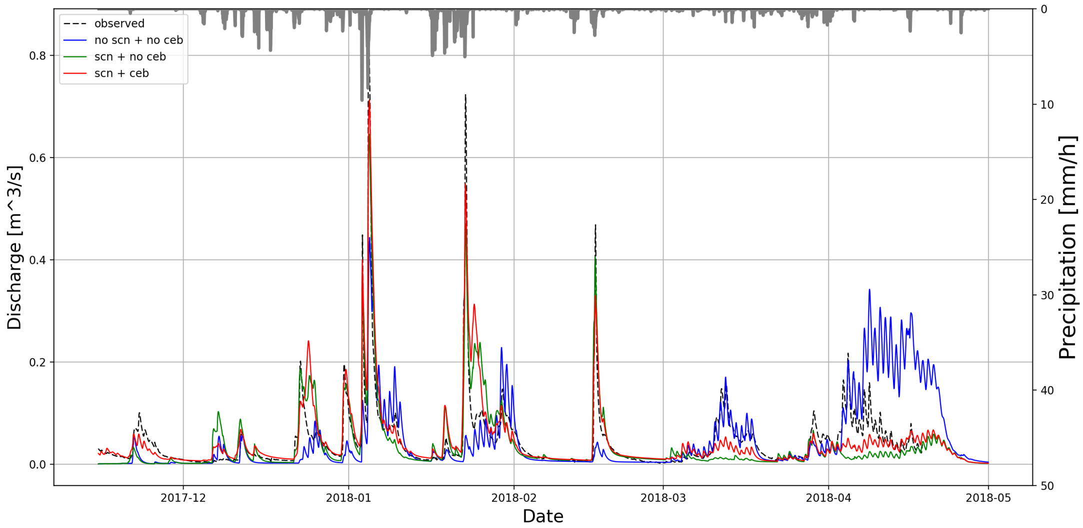

Figure 3: Comparison of simulated and measured discharge data during validation run in winter 2017/18

It can easily be recognized that the parameterization of model 1 fails to reproduce the discharge behaviour during the validation period. Taking into account the interception processes leads to a significant improvement of the model performance. The use of the CEB further improves the model quality. Table 1 gives an overview of different model efficiency coefficients for the three different models during the validation.

Table 1: Comparison of different model efficiencies for the validation run

	NSE [-]	logNSE [-]	PBIAS [%]
No SCN + no CEB	0.63	0.43	41.0
SCN + no CEB	0.68	0.69	20.2
SCN + CEB	0.79	0.76	0.1

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