Dynamic time step control

The implementation of a dynamic time step controller into the Richards-version of WaSiM allows a more flexible and process-oriented soil parametrization as well as an improved simulation quality of vertical water fluxes in layered soils compared to a uniform internal time step. Using this controller, soil water fluxes can be simulated with a very high level of numerical stability even in cases with extreme parameter settings (e.g. very thin soil layers combined with the pronounced hydraulic conductivity of sandy soils).

The introduced dynamic time step control algorithm (available in WaSiM since version 7.9) considers explicitly the Courant stability criterion in order to prevent oscillations possibly caused by the discretization scheme of the actual soil profile. The Courant criterion is defined as follows:

$$Co = \left| \frac{\Delta t \cdot v}{\Delta x} \right| \leq 1 \qquad \textbf{(2.14.45)}$$

with Co Courant criterion
Δt time step [s]
v velocity [m/s]
Δx spatial distance between the nodes [m]

After its transformation the boundary condition for a stable numerical solution are given by:

$$\Delta x \geq \Delta t \cdot v \quad \text{resp.} \quad \Delta t \leq \frac{\Delta x}{v} \qquad \textbf{(2.14.46)}$$

with Δt simulation time step [s]
v hydraulic conductivity [m/s]
Δx thickness of the actual soil layer [m]

In WaSiM the algorithm of dynamic time step control has been a slightly adapted:

$$\Delta t = MIN_{i=1}^{k} \left(\frac{\Delta z_i}{k(\Theta)_{rel,i} \cdot k_{sat,i}} \right) \qquad \textbf{(2.14.47)}$$

with Δt minimum time for flowing of water through the soil layer at the actual water content [s]
i layer index (from 1..k)
k index of last soil layer above the groundwater level
Δz_i thickness of the actual soil layer i [m]
$k(\Theta)_{rel,i}$ relative moisture-dependent conductivity of layer i
$k_{sat,i}$ saturated hydraulic conductivity of layer i [m/s]

If the Courant time interval is shorter than the predefined model time step ($\Delta t < I$) then a certain split factor has to be calculated in order to reduce the length of the original time step:

$$f_{split} = \left\lceil \frac{I}{\Delta t} \right\rceil \qquad \textbf{(2.14.48)}$$

with f_{split} split factor with $f_{split} >= 1$
I length of the selected time step interval (as defined in control file) [s]
Δt result from Equation (2.14.11) [s]
$\lceil\rceil$ character for a rounded up integer

The internal reduction of the (original) model time step is done separately for each grid cell. For this purpose the implemented control algorithm checks all soil layers above the groundwater level with regard to the actual Courant criterion. If necessary ($\Delta t < I$) then the (original) model time step is subdivided into a certain number of subintervals (f_{split}). In this case, the absolute input data [in mm] for the soil model have to be recalculated for each subinterval whereas data of flow velocity (e.g. descent and ascent rates of the groundwater level or water flows between soil layers) can be used as before (i.e. without subdivision) because of their independence of the length of calculation time interval.

The shortest possible subinterval is one second. That means in effect that a model time step of one hour can be subdivided into up to 3600 subintervals. In order to increase the flexibility of the implemented algorithm, it is possible to define a minimum subinterval length [in s].

The dynamic time step control can be activated by selecting method “3” in the section [unsatzon_model] of the WaSiM control file.